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Moduli Potentials in type-IIA compactifications with RR and NS flux

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Abstract: We describe a simple class of type-IIA string compactifications on Calabi-Yau manifolds where background fluxes generate a potential for the complex structure moduli, the dilaton, and the Kähler moduli. This class of models corresponds to gauged $\mathcal{N} = 2$ supergravities, and the potential is completely determined by a choice of gauging and by data of the $\mathcal{N} = 2$ Calabi-Yau model — the prepotential for vector multiplets and the quaternionic metric on the hypermultiplet moduli space. Using mirror symmetry, one can determine many (though not all) of the quantum corrections which are relevant in these models.

Keywords: Superstring Vacua, Supergravity Models, Supersymmetric Effective Theories.

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1. Introduction

Finding compactifications with computable potentials for the scalar moduli is an important problem in string theory. Early ideas in this direction, in the context of the heterotic string, can be found in [1, 2]. More recently, there has been a great deal of activity exploring the potentials generated by p-form fluxes in type II string models (for some excellent reviews with references, see e.g. [3, 4]). Much of the attention has been focused on type-IIB Calabi-Yau flux vacua, where the flux-induced potentials depend on the complex and dilaton moduli, but not the Kähler moduli [5, 6]. In this type-IIB context, there is increasingly strong evidence that proper incorporation of quantum corrections (to the superpotential and/or Kähler potential) yields large numbers of models where the Kähler moduli can be stabilized as well [7 − 11]. Concrete examples of moduli stabilization which work outside the framework of low-energy supersymmetry have also been developed [12]. In the present paper, we describe a class of type-IIA flux vacua based on Calabi-Yau compactification, where the fluxes alone generate a potential for all geometrical moduli.

One of the basic difficulties with analyzing $\mathcal{N} = 1$ string compactifications is that the scalar potential receives corrections at all orders in α' and g_s. Since one is usually ignorant of the exact Kähler potential K, i.e. the exact two-derivative lagrangian, corrections to the potential which are suppressed by the ratio of the SUSY breaking scale M^2 to M_s^2 and which are difficult to compute will typically arise. For instance, in a background where a chiral field ϕ has a SUSY-breaking auxiliary field VEV $F_\phi \neq 0$, one can potentially soak up the superspace θ integrals in $\int d^4\theta\, K$ by using F_ϕ, and hence terms in K proportional to $\phi^\dagger\phi$ can correct the scalar potential V. This is not necessarily a problem, since in models where one obtains moderately weak couplings by tuning, such corrections can be controlled (as in standard perturbative quantum field theory) — this is the situation for many of the proposed string constructions with stabilized moduli. Nevertheless, this does represent a concrete limitation on one's knowledge of the potential.

In the class of models we describe here, in contrast, one can hope to compute the exact two-derivative lagrangian (though it is still a highly nontrivial task). These models are based on $\mathcal{N} = 2$ gauged supergravities; while such models cannot be completely realistic, they can be useful toy models for the more general $\mathcal{N} = 1$ situation. Among the many developments during the duality revolution of the mid 90s was the discovery that $\mathcal{N} = 2$ supersymmetric string vacua are in some sense exactly soluble [13]. Using heterotic/type-II duality as well as mirror symmetry, one can find the exact prepotential for the $\mathcal{N} = 2$ vector multiplets at string tree-level in the type-II picture, and the geometry of the quaternionic manifold at string tree-level in the heterotic picture. In practice this can be carried out for the vector multiplets in simple examples (see e.g. [14] where the Seiberg-Witten solution of $\mathcal{N} = 2$ gauge theory [15] was recovered using these dualities). On the other hand, it has proved dauntingly difficult to understand the geometry of hypermultiplet moduli spaces (see e.g. [16] for some attempts). This is partially because in the IIB picture the vector multiplet moduli space is exact at both string and sigma model tree-level, while even in the heterotic picture the quaternionic manifold receives corrections in sigma model perturbation theory (though not from string loops). Nevertheless, $\mathcal{N} = 2$ vacua are clearly under better control than their $\mathcal{N} = 1$ counterparts.

In this paper, we describe a class of type-IIA Calabi-Yau compactifications where a potential for all geometrical moduli (as well as the axio-dilaton) is determined completely by the $\mathcal{N} = 2$ prepotential and quaternionic metric, as well as a choice of gauging. In string theory terms, these models arise from IIA compactifications on Calabi-Yau spaces with the RR four-form flux F_4 and the NS three-form flux H_3 turned on. While the relevance of gauged supergravity to flux potentials in string theory has been discussed extensively [17–21], most of the compact Calabi-Yau flux models which have been constructed to date also include orientifold planes. The presence of the orientifold planes breaks the supersymmetry to $\mathcal{N} = 1$, and so the results derived from the gauged supergravity analysis are not exact (though they can be an excellent approximation). In particular, the Kähler potential need not be the one which follows from gauged supergravity. In our class of models, the results

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from gauging (in terms of the fully corrected prepotential and quaternionic metric) should be an even better approximation to the full theory. Since in the $\mathcal{N}=2$ models we know the precise Kähler potential, one expects the first corrections to the potential that we are neglecting to be down by two additional powers of M_s, in comparison with typical $\mathcal{N}=1$ models.

Our models are simpler, more computable relatives of G_2 flux compactifications of M-theory. Models with only four-form flux turned on in that context do not yield vacua in the large volume approximation, a fact we will see reflected here as well; orientifolds of our IIA models should provide insight into quantum corrections which generate more structure in the effective potential in such compactifications. Earlier suggestions for moduli stabilization in M-theory compactifications appeared in [22]. Some interesting Calabi-Yau compactifications of IIA strings with only RR flux turned on were considered in [17], while a different class of type-IIA compactifications with flux was described in [23].

The organization of this paper is as follows. In section 2, we describe the general gauged supergravity framework which encompasses our models. In section 3, we present a toy example which shows that the resulting potentials can have interesting features. We close with a discussion in section 4. Our calculations have been relegated to several appendices.

2. Scalar potentials from gauging

In this section, we describe the scalar potentials which arise in IIA Calabi-Yau compactifications with RR four-form flux (and six-form flux with qualifications, see below) and NS three-form flux turned on in the internal dimensions. In section 2.1, we describe how the potentials are derived given the data of a 4d $\mathcal{N}=2$ supersymmetric effective field theory. In section 2.2 and section 2.3, we describe how IIA string compactifications on Calabi-Yau threefolds with nontrivial F_4 and H_3 give examples of the class of theories described in section 2.1. The latter analysis relies heavily on earlier work of Louis and Micu [19].

2.1 Scalar potentials from $\mathcal{N}=2$ data

The data of an $\mathcal{N}=2$ supergravity theory in four dimensions includes a special Kähler manifold $\mathcal{M}_V$, the moduli space of vector multiplets, and a quaternionic Kähler manifold $\mathcal{M}_H$, the moduli space of hypermultiplets. In type-IIA compactification on a Calabi-Yau space X, these correspond to the Kähler moduli space and the complex structure + axio-dilaton moduli space respectively (with the latter enhanced to a quaternionic manifold by the presence of the RR axions).

The geometry of $\mathcal{M}_V$ can be described by complex projective coordinates X^I, $I = 0,\ldots,h^{1,1}(X)$, and a prepotential F. The Kähler potential on $\mathcal{M}_V$ is

$$K = -\log[i(\overline{X}^I\mathcal{F}_I - X^I\overline{\mathcal{F}}_I)]\,. \tag{2.1}$$

$\mathcal{M}_H$ comes equipped with a quaternionic metric h_{uv}, where $u,v = 1,\ldots,4(h^{2,1}(X)+1)$.

In an $\mathcal{N}=2$ gauged supergravity [24], we in addition choose Killing vectors k_I^i and k_I^u which generate the action of the Ith gauge field on the vector and hypermultiplet moduli ($I=0$ corresponds to graviphoton charges; we also note that for abelian gauging, the case we will be considering in this paper, k_I^i of course vanish). These are related to an SU(2) triplet of Killing prepotentials $\mathcal{P}_I^x$ (for $x=1,2,3$). The scalar potential is given in terms of the Killing vectors and the Killing prepotentials by

$$V = e^K X^I \bar{X}^J (g_{i\bar{j}} k_I^i k_J^{\bar{j}} + 4h_{uv} k_I^u k_J^v) - \left(\frac{1}{2}(\mathrm{Im}\,\mathcal{N})^{-1\,IJ} + 4e^K X^I \bar{X}^J\right)\mathcal{P}_I^x \mathcal{P}_J^x \,. \tag{2.2}$$

$\mathrm{Im}\,\mathcal{N}$ is the gauge coupling matrix,

$$\mathcal{L}_{kin}^{vec} = i(\bar{\mathcal{N}}_{IJ} F^{-I} \wedge *F^{-J} - \mathcal{N}_{IJ} F^{+I} \wedge *F^{+J}) \tag{2.3}$$

$$= (\mathrm{Im}\,\mathcal{N})_{IJ} F^I \wedge *F^J - i(\mathrm{Re}\,\mathcal{N})_{IJ} F^I \wedge F^J \,. \tag{2.4}$$

The important point for us is the following: the scalar potential of the resulting theory is completely determined in terms of a choice of isometries and the data characterizing $\mathcal{M}_V$ and $\mathcal{M}_H$. Hence, although it is difficult work to compute the low-energy effective action of the $\mathcal{N}=2$ theory resulting from Calabi-Yau compactification, this action together with various choices of charges (fluxes) completely determines the potential (2.2) in this class of models.[1]

Most Calabi-Yau flux compactifications to date have involved, in addition to fluxes, further explicit breaking of the supersymmetry (or have involved highly simplified models, like toroidal or $K3$ orientifolds). In section 2.3, we describe a class of IIA Calabi-Yau compactifications where the theory is really an $\mathcal{N}=2$ gauged supergravity, and the formula (2.2) is the full result for the potential.

2.2 Corrections to the tree level CY potential

It is a celebrated result in (ungauged) $\mathcal{N}=2$ supergravity that up to the two derivative level, no interaction terms can be introduced that involve both vector multiplets and (neutral) hypermultiplets. For CY string compactifications, this implies that the metric on the vector multiplet scalar manifold receives no string loop corrections, and the metric on whichever scalar manifold coincides with the complex structure moduli space of the CY at tree level (hyper for IIA, vector for IIB) receives no α' corrections. In gauged supergravity, hypers can acquire charges under vectors. The two sectors of the theory are then of course no longer decoupled. Nevertheless, the above conclusions can still be drawn: the structure of the lagrangian is completely encoded in terms of the same type of $\mathcal{N}=2$ data as before gauging, i.e. the metric on a special Kähler manifold for the vectors and on a quaternionic manifold for the hypers (in addition, as pointed out above, killing vectors encoding the

[1] It could be that at the level of e^{-1/g_s} corrections, the relevant metric on $\mathcal{M}_H$ is changed by the presence of fluxes. This is particularly plausible because euclidean D2 branes which give rise to such corrections, cannot wrap cycles threaded by NS three-form flux. The expected size of the e^{-1/g_s} corrections can be made negligibly small by tuning g_s as we describe explicitly in section 3.1, and hence these effects play no significant role in our considerations.

isometries of the scalar manifolds that are to be gauged must be specified) [24]. As before, the presence of a potential notwithstanding, no mixing of the metrics is allowed, and the same exactness conclusions as above can then be drawn.

In the presence of the fluxes, backreaction corrects the Calabi-Yau geometry and it is no longer a solution of the equations of motion. However, there is strong evidence that the result of turning on fluxes in the Calabi-Yau must, at the level of 4d effective field theory, simply be to gauge the $\mathcal{N}=2$ supergravity arising from the model with no flux. We assume this to be true (up to the possible caveat about e^{-1/g_s} corrections to the metric on $\mathcal{M}_H$ mentioned above). Then, the powerful results of [24] allow us to write down the 4d effective field theory resulting from the corrected geometry in terms of the $\mathcal{N}=2$ data of the fluxless model . Therefore, even without detailed knowledge of the structure of the corrected supergravity or string solution, we are able to discuss the effective potential of the resulting low-energy theory in 4d with confidence.

It is important to keep in mind that the "exact" results from gauged $\mathcal{N}=2$ supergravity receive corrections when one expands the resulting potentials around critical points which break supersymmetry. In any systematic attempt to make controlled examples, the relevant parameter controlling corrections will be $\mu = M/M_s$ where M is the supersymmetry breaking scale. Since the first unknown corrections will enter at higher orders in the μ expansion for $\mathcal{N}=2$ gauged models than for general $\mathcal{N}=1$ models, we expect these models to be a good laboratory for studying the space of vacua.

2.3 IIA strings with F_4 and H_3 flux

The class of models we will consider differ from the hitherto popular IIB models in two main regards. Unlike the situation in IIB, we demonstrate below that tadpole constraints do not force us to break $\mathcal{N}=2$ SUSY explicitly, e.g. by orientifolding, once we turn on fluxes in IIA. This is why we can extend the power of $\mathcal{N}=2$ beyond the traditional scenario without fluxes. Also, in IIA compactifications on CYs, RR and NS fluxes thread cycles of different dimensions. Since the size of even dimensional cycles is controlled by the Kähler data of the geometry, and the size of middle dimensional cycles is controlled by the complex structure moduli, turning on both types of fluxes gives rise to non-trivial dependence on both complex structure and Kähler moduli in the potential already at the perturbative level in g_s.[2]

2.3.1 Tadpole constraints

In type-IIB, turning on both RR and NS 3-form flux F_3 and H_3 gives rise to a D3 brane tadpole visible in the CS term

$$\int C_4 \wedge H_3 \wedge F_3 \tag{2.5}$$

in the SUGRA action. We wish to determine whether such tadpoles can arise in IIA. The fluxes in the game are F_0, F_2, F_4, F_6 and H_3. Only tadpoles for space filling branes lead to

[2]The 'non-trivial' dependence is to be contrasted with the no-scale potentials in IIB [6], which also have dependence on both types of moduli, the Kähler moduli however only entering trivially via the e^K prefactor in the potential of $\mathcal{N}=1$ supergravity.

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inconsistencies due to violation of Gauss' law in the compact space. Hence, we only need to worry about the gauge potentials C_5, C_7, C_9.

- A C_5 tadpole could only arise from a term $\int C_5 \wedge F_2 \wedge H_3$. Since a CY has no non-trivial 5 cycles, such a term cannot arise in a CY compactification.
- A C_7 tadpole can arise from a term $\int C_7 \wedge H_3$. Such a term arises from the kinetic term of $\tilde{F}_2 = dC_1 + mB_2$ in massive IIA, which is IIA with F_0 flux turned on,

$$\begin{aligned}\int \tilde{F}_2 \wedge *\tilde{F}_2 &\to m \int B_2 \wedge *dC_1 && (2.6)\\ &= m \int B_2 \wedge dC_7 && (2.7)\\ &= -m \int H_3 \wedge C_7 \,. && (2.8)\end{aligned}$$

- Finally, there are no 1-forms present to give rise to a C_9 tadpole.

The only tadpole for space filling branes that can be generated in this setup hence arises in the presence of both F_0 and H_3 flux. It is a C_7 tadpole, corresponding to a space filling D6 brane wrapping a 3 cycle in the CY. We will avoid this tadpole by simply leaving the RR F_0 flux turned off.

2.3.2 The Chern-Simons term

Before proceeding with our analysis, we need to address a subtlety. Upon turning on fluxes, the relation between field strength and gauge potential, $F = dA$, no longer holds. In particular, various incarnations of the CS terms that are related by integration by parts before turning on fluxes are no longer equivalent. In [19], this issue is dealt with pragmatically by choosing a form of the action in which the compromised gauge potentials do not appear explicitly in the CS term. This is no longer possible once both RR and NS fluxes are turned on, at least not in 10 dimensions.[3]

Requiring that the 10 dimensional action upon compactification fits into the constraining harness of $\mathcal{N} = 2$ gauged SUGRA, we arrive at the following proposal for the 10d CS term in a Calabi-Yau background in the presence of fluxes:

$$S_{\rm CS} = \int \frac{1}{2}(dB + H_3^{\rm flux}) \wedge C_3 \wedge dC_3 - B \wedge F_4^{\rm flux} \wedge dC_3 \,. \tag{2.9}$$

This term clearly reduces to the standard CS term in the absence of fluxes. To derive this proposal from first principles, one could, in the spirit of [25], introduce a CS term on an 11-manifold $\mathcal{W}$ that has the physical 10 dimensional space $\mathcal{M}$ as boundary, $\partial\mathcal{W} = \mathcal{M}$,

$$S_{\rm CS} = -\frac{1}{2} \int_{\mathcal{W}} H_3 \wedge F_4 \wedge F_4 \,. \tag{2.10}$$

[3] Any fears that turning on RR and NS flux simultaneously may not be consistent should be alleviated, at least for the case of turning on both F_4 and H_3, by the fact that this situation arises upon compactification of M-theory on a circle in a generic G-flux background.

Assuming that $\mathcal{M}$ is of the form $X \times Z$ where X is a Calabi-Yau manifold and Z represents the noncompact dimensions, and that the cohomology of the Calabi-Yau (together with its ring structure) is preserved in $\mathcal{W}$, this prescription gives rise to the 10d CS term (2.9) proposed above. To elevate this heuristic sketch into a derivation, one must show that appropriate manifolds $\mathcal{W}$ exist, and that the 10d term is independent of which manifold $\mathcal{W}$ one chooses. We take the fact that (2.9) reduces to previously proposed formulae in suitable limits, together with the fact that it gives rise to a dimensional reduction which fits into the expected supergravity framework, to be sufficient evidence for our proposal, and leave a more rigorous argument (perhaps along the lines suggested above) for future work.

2.3.3 Gauged supergravities arising from F_4 and H_3 flux

The fluxes we have at our disposal are F_0, F_2, F_4, F_6 and H_3. To avoid the need to cancel a $D6$ brane tadpole, we set $F_0 = 0$. As Louis and Micu [19] have demonstrated (in the absence of H_3 flux), turning on F_2 and F_4 simultaneously gives both electric and magnetic charges to the axion a (the $\mathcal{N} = 1$ SUSY partner of the dilaton) under the gauge fields in the vector multiplet. We bypass such complications by also setting $F_2 = 0$ (we could just as well consider turning on F_2 flux and setting $F_4 = 0$; this merely swaps electric for magnetic charges for the RR axions). By $F_6 = *F_4$, turning on F_6 flux is equivalent to modifying the *spacetime* part of F_4. Hence, the presence of F_6 flux can be dealt with *after* performing the dimensional reduction. We are thus left with reducing in the presence of F_4 and H_3 flux. We give the details of this calculation in appendix A.1. Here, we state our results.

We consider compactification on a Calabi-Yau X with the fluxes

$$F_4^{\text{flux}} = e_i \tilde{\omega}^i \,, \tag{2.11}$$
$$H_3^{\text{flux}} = p^A \alpha_A + q_A \beta^A \tag{2.12}$$

turned on, where $\tilde{\omega}^i$, $i = 1, \ldots, h_{1,1}$, are a dual basis for $H^{1,1}(X)$,[4] and α_A, β^A, $A = 0, \ldots, h_{2,1}$, a symplectic basis for $H^3(X)$. In this flux background, the axion a as well as the RR axions $\xi^A, \tilde{\xi}_A$ become charged under the graviphoton. In addition, a also acquires charges under all vector multiplets. Specifically, the killing vectors are given by

$$\begin{aligned} k_0^a &= 2n - 2b^i e_i + p^A \tilde{\xi}_A - q_A \xi^A \,, \qquad (2.13)\\ k_0^{\xi^A} &= p^A \,, \\ k_0^{\tilde{\xi}_A} &= q_A \,, \\ k_i^a &= -2e_i \,, \qquad (2.14) \end{aligned}$$

where $n \in \mathbb{Z}$ can be interpreted as F_6 flux (see A.1). The b^i are the partners of the (metric) Kähler moduli on the complexified Kähler cone. As expected, the isometries being gauged by the graviphoton and the gauge fields of the vector multiplets commute pairwise. Comparing to [19], we see that the most naive assumption holds true: the isometries being gauged upon turning on both RR and NS flux are simply the sum of those gauged upon turning on the fluxes individually.

[4] Meaning, if we take ω_i to be a basis for $H^{1,1}(X, \mathbb{Z})$, then $\int_X \omega_i \wedge \tilde{\omega}^j = \delta_i^j$.

The potential we obtain from dimensional reduction also follows, as required for consistency, from the general form of the gauged $\mathcal{N}=2$ potential (2.2) with the above choice of killing vectors. It is given by

$$V=\frac{e^{4\phi}}{2\mathcal{K}}\left(\frac{1}{4}g^{ij}e_ie_j+(n-b^ie_i+p^A\tilde{\xi}_A-q_A\xi^A)^2\right)-\frac{e^{2\phi}}{4\mathcal{K}}(q+p\mathcal{M})(\operatorname{Im}\mathcal{M})^{-1}(q+p\bar{\mathcal{M}})\,, \quad (2.15)$$

where $\mathcal{K}=\frac{1}{8}e^{-K}$ is the volume of X and $g_{ij}=\frac{1}{4\mathcal{K}}\int_X\omega_i\wedge *\omega_j$. Here $\mathcal{M}$ is the matrix defined by

$$\mathcal{M}_{AB}=\overline{\mathcal{G}}_{AB}+2i\frac{(\operatorname{Im}\mathcal{G})_{AC}Z^C(\operatorname{Im}\mathcal{G})_{BD}Z^D}{Z^C(\operatorname{Im}\mathcal{G}_{CD})Z^D}\,, \quad (2.16)$$

where Z^A and $\mathcal{G}_A$ are the periods and dual periods in a symplectic basis for H^3, and further subscripts on $\mathcal{G}$ indicate differentiation with respect to the relevant complex modulus.

In this section we have simply summarized the results of our computations because they are somewhat lengthy and involved. The interested reader can find the derivation of the potential (2.15) from dimensional reduction in appendix A.1, and from the general form of the $\mathcal{N}=2$ gauged supergravity potential (given the killing vectors (2.13)) in appendix A.2.

3. Worldsheet instantons and a toy example

One class of minima of the potential (2.15) lie at infinite Kähler parameter. This comes as no surprise: whenever the Kähler class dependence is purely that of classical geometry, fluxes will drive the geometry to large Kähler class, as this causes them to be diluted and reduces their contribution to the energy. However, as described in section 2.2, we need not restrict our attention to the large volume limit; the data relevant for computing the scalar potential (in particular, the $\mathcal{N}=2$ prepotential) is available for all values of the Kähler class. So, we can simply use the full instanton corrected $\mathcal{N}=2$ data of the CY model, to compute the scalar potential (2.2) including worldsheet instanton corrections. While this expression for the potential is not exact, the first corrections we are neglecting are suppressed by more powers of M_s than in the analogous $\mathcal{N}=1$ constructions. Hence, in any case where the SUSY breaking order parameters are smaller than M_s, corrections may be controllable.[5]

We leave the construction of such classes of examples for future work, and here provide only a simple illustrative model that demonstrates that our potentials can contain interesting structure. For our toy example, we will join together the complex structure of a rigid CY with the Kähler moduli space of a one Kähler parameter CY. Our motivation for considering this fictional geometry is to isolate the main features of this class of compactifications without being swamped by too many computational hurdles. Such a geometry would give rise to one 4 dimensional vector multiplet and one (the universal) hypermulti-

[5]We expect that it will probably be easier to construct families of such examples in orientifolds of our class of models, where the negative term in the potential coming from the O-plane tensions plays a helpful role in stabilizing g_s and volume moduli; for a nice discussion of this in component form see [3].

plet. With the killing vectors determined in the previous section and the associated killing prepotentials determined in appendix B, the potential for our toy example takes the form

$$V = -e^{4\phi}(\mathrm{Im}\,\mathcal{N})^{-1\,IJ}a_I a_J - \frac{1}{2}e^{2\phi}(\tilde{p}^2+\tilde{q}^2)\left((\mathrm{Im}\,\mathcal{N})^{-1\,00} + \frac{3}{4\mathcal{K}}X^0\bar{X}^0\right), \tag{3.1}$$

where $a_0 = \frac{1}{\sqrt{2}}n + \tilde{p}y - \tilde{q}x$ and $a_1 = \frac{1}{2\sqrt{2}}e_1$. We have here rotated the RR axions and the fluxes to the more convenient basis $\sqrt{2}\binom{x}{y} = \begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}\binom{\xi}{\tilde{\xi}}$, $\frac{1}{\sqrt{2}}\binom{\tilde{p}}{\tilde{q}} = \begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}\binom{p}{q}$, where the transformation matrix is a real symplectic matrix whose entries depend on the period matrix of the complex structure moduli space. The explicit entries are given in appendix B.

3.1 The moduli of the universal hypermultiplet

The dependence of the potential on the scalar fields of the hypermultiplet is very simple. The potential is quadratic in the string coupling and quadratic in the RR axions, which occur only in the combination $a_0 = \frac{1}{\sqrt{2}}n + \tilde{p}y - \tilde{q}x$. There is no dependence on the axion a.

The dilaton: highlighting the dilaton dependence, the potential takes the form

$$V = e^{2\phi}(A_2 e^{2\phi} + A_1)\,, \tag{3.2}$$

where

$$A_1 = -\frac{1}{2}(\tilde{p}^2+\tilde{q}^2)\left((\mathrm{Im}\,\mathcal{N})^{-1\,00} + \frac{3}{4\mathcal{K}}X^0\bar{X}^0\right) \tag{3.3}$$

$$A_2 = -(\mathrm{Im}\,\mathcal{N})^{-1\,IJ}a_I a_J\,. \tag{3.4}$$

For fixed A_1 and A_2, there is hence always a stationary point at vanishing string coupling ($g_s = e^{\phi}$), where the potential and all of its derivatives with regard to ϕ vanish. For positive A_1, this point is a minimum. The other stationary point lies at $e^{2\phi} = -\frac{A_1}{2A_2}$. Since the dilaton is a real scalar field (and hence the string coupling is always real and positive), this is only a physically acceptable solution when the r.h.s. of this equation is positive.

The analysis thus hinges on the signs of A_1 and A_2. Let us consider these in turn. $\mathrm{Im}\,\mathcal{N}$ is the gauge coupling matrix, and hence negative definite (negative rather than positive definite due to its standard definition in SUGRA). A_2 is therefore always positive. For the potential to have a minimum away from vanishing string coupling $e^{\phi} = 0$, A_1 must thus be negative. This imposes the following inequality on the $\mathcal{N}=2$ data:

$$4(\mathrm{Im}\,\mathcal{N})^{-1\,00}\mathcal{K} + 3X^0\bar{X}^0 > 0\,. \tag{3.5}$$

Note that the choice of the values of the fluxes did not enter into these considerations. This is a simplification that would not persist in more general models, but arises because of our extremely simple choice of vector and hyper moduli spaces.

In the classical limit, in the gauge $X^0 = 1$, $(\mathrm{Im}\,\mathcal{N})^{-1\,00}\mathcal{K} = -1$, and the inequality is not satisfied. To move into the interior of moduli space, we need to pick a model. We will do so below. First, let us assume that we can find a model with $A_1 < 0$ in some region of the Kähler moduli space, and press on with the analysis. Plugging in the functional dependence of the string coupling on the remaining parameters of the theory at the minimum, we obtain the following expression for the potential:

$$V_\phi = -\frac{A_1^2}{4A_2}\,. \tag{3.6}$$

The logic of the notation is that the subscript denotes the field that has been eliminated from the potential.

The RR axions: V_ϕ depends on the RR axions x and y in the combination $a_0 = \frac{1}{\sqrt{2}}n + \tilde{p}y - \tilde{q}x$. Minimizing with regard to a_0 yields the potential

$$V_{\phi,\xi,\tilde{\xi}} = \frac{1}{4}\frac{A_1^2(\mathrm{Im}\,\mathcal{N})^{-1\,00}}{a_1^2\det\mathrm{Im}\,\mathcal{N}^{-1}} \tag{3.7}$$

$$= \frac{1}{256}\frac{(4(\mathrm{Im}\,\mathcal{N})^{-1\,00}\mathcal{K}+3X^0\bar{X}^0)^2(\mathrm{Im}\,\mathcal{N})^{-1\,00}}{\mathcal{K}^2\det\mathrm{Im}\,\mathcal{N}^{-1}}\frac{(\tilde{p}^2+\tilde{q}^2)^2}{a_1^2}\,. \tag{3.8}$$

Since the eigenvalues of the gauge coupling matrix are, away from singular points, always negative, the determinant is strictly positive. The sign of $V_{\phi,\xi,\tilde{\xi}}$ hence depends on the sign of $(\mathrm{Im}\,\mathcal{N})^{-1\,00}$. A simple argument shows that in models with a single Kähler parameter, this must be negative: at large radius, $(\mathrm{Im}\,\mathcal{N})^{-1\,00}$ is negative. If it is positive at some point in moduli space, it must, by continuity and the fact that singular points on the moduli space are of complex codimension one, vanish at some point. We know that the determinant of the gauge coupling matrix is strictly positive away from singular points. The matrix being symmetric, the off diagonal elements contribute negatively to the determinant. Hence, neither of the diagonal elements can vanish anywhere on moduli space, and they must therefore retain their sign throughout moduli space. We conclude that $V_{\phi,\xi,\tilde{\xi}}$ for our toy example will vanish or be negative. Any minimum will hence be anti de Sitter, in this approximation. Furthermore, the potential is bounded from below[6] and vanishes in the large radius limit. So we can conclude that as long as there is some region in Kähler moduli space where the potential goes negative (as happens if $A_1 < 0$), it must attain a minimum.

Once we minimize with regard to a_0, we obtain the following flux dependence for the string coupling

$$g_s^2 \sim \frac{\tilde{p}^2+\tilde{q}^2}{e_1^2}\,. \tag{3.9}$$

By a judicious choice of fluxes, one can therefore find vacua at weak string coupling. This feature will persist in more general models; in the limit where the RR flux quantum numbers are larger than the NS flux quantum numbers, the coupling will always be weak.

3.2 The Kähler moduli

The dependence on Kähler moduli enters the potential via the gauge coupling matrix and X^0. For our toy example, we choose the Kähler moduli space as that of the sextic in

[6]The only singular point on moduli space is the (mirror) conifold point, and $V \to 0$ there as g_s is driven to vanish. At any finite value of g_s, the potential (naively) tends to positive infinity there. This is related to the fact that the dilaton carries electric charge, in contrast to the situation studied in [17]. The potential is manifestly finite at any smooth points.

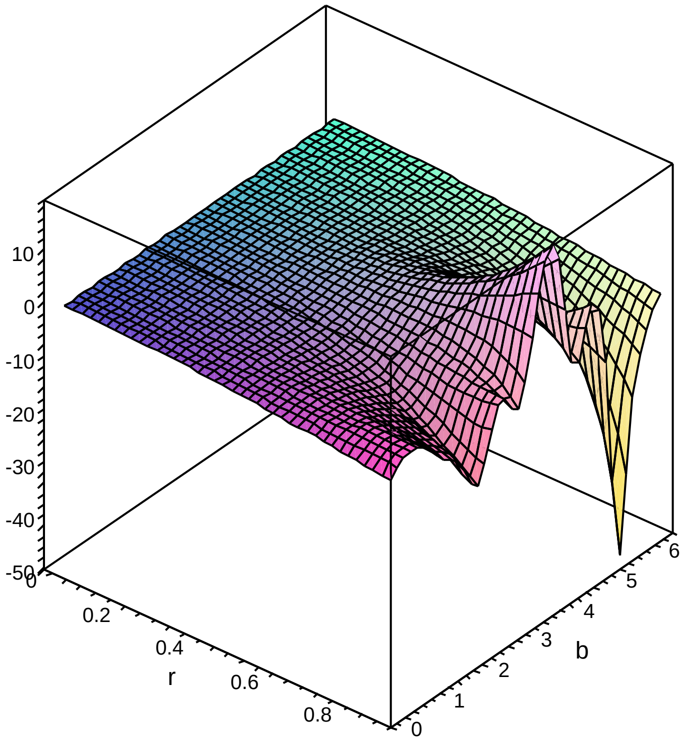

Figure 1: A plot of $4(\mathrm{Im}\,\mathcal{N})^{-1\,00}\mathcal{K} + 3X^0\bar{X}^0$ against ψ. To produce this and the following plot, the periods around the LG point were expanded to order 6 in ψ.

$W\mathbb{P}^4_{2,1,1,1,1}$. Using mirror symmetry, this is equivalent to the complex structure moduli space of the orbifold of the hypersurface $W = 2x_0^3 + x_1^6 + x_2^6 + x_3^6 + x_4^6$ in $W\mathbb{P}^4_{2,1,1,1,1}$ by the group $G = \mathbb{Z}_3 \times \mathbb{Z}_6^2$ [29, 30]. Following the calculational steps outlined in appendix C, we obtain the $\mathcal{N} = 2$ data necessary to write down the potential in an expansion around the Landau-Ginzburg point.

As outlined above, the crucial step in our analysis is finding a region in moduli space in which the inequality (3.5) is satisfied, such that $A_1 < 0$ and the minimum of the potential lies away from vanishing string coupling. This is equivalent to requiring that $(\mathrm{Im}\,\mathcal{N})^{-100} + \frac{3}{4\mathcal{K}}X^0\bar{X}^0 > 0$. A plot of the l.h.s. of this inequality over the ψ plane, where $\psi = re^{ib}$ is the coordinate on the Kähler moduli space in a neighborhood of the Landau-Ginzburg point (see C.1), is shown in figure 1.

Computing (3.2) precisely at the Landau-Ginzburg point, we see that the minimum of the potential in the dilaton direction would arise by taking $g_s \to 0$ (where V vanishes). On the other hand, as we vary the expectation value of the Kähler mode by hand, we see that for $r \sim 0.6$ and a range of values of the axion, the dilaton vacuum arises at a finite value of g_s. At this critical point $V < 0$. As explained in section 3.1, this is all we need to know to infer the existence of a minimum of the full potential at finite r. To examine whether this minimum lies within our range of computability, we consider a plot of $V_{\phi,\xi,\tilde{\xi}}$ in figure 2. The plot suggests that a minimum of the potential lies very close to $r = 1$, (but at $b \sim 3.7$, i.e. far from the conifold). This result should be verified by expanding

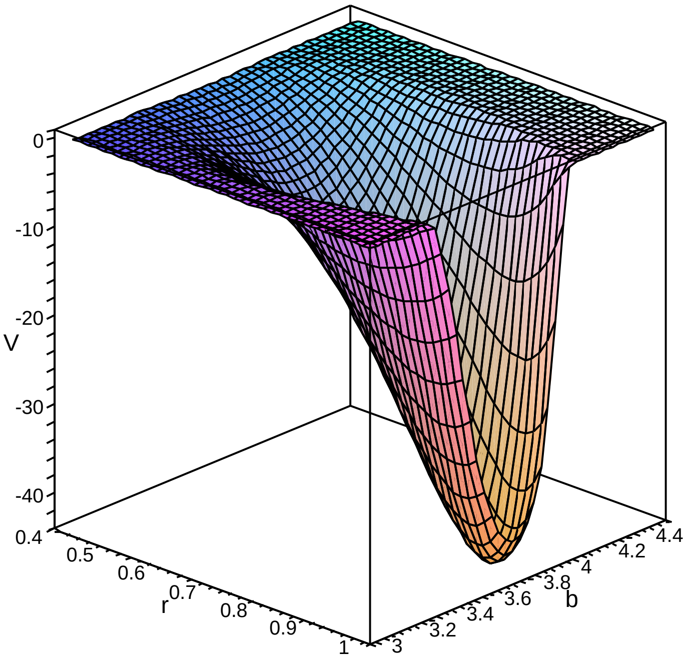

Figure 2: A plot of $V_{\phi,\xi,\tilde{\xi}}$ against ψ.

the periods to higher order in ψ. Again, we can infer the *existence* of a minimum simply by observing the potential take on negative values, and this happens already at $r \sim 0.6$, where we trust our calculation.

Note that in computing the potential, we allow the NS axion partner of the Kähler mode to vary over a full 2π period; this results in a smooth potential. While one might naively expect the axion to vary only over a pie-wedge of angle $\frac{2\pi}{6}$ in this model, the fluxes spontaneously break the $\mathbb{Z}_6$ symmetry responsible for this identification. Hence upon leaving one pie-wedge and entering the neighboring region, one can either perform a modular transformation (determined by the LG monodromy) which changes the fluxes (and is necessary to obtain a smooth potential), or one can fix the fluxes and allow the axion to vary over a larger region in field space. We choose the latter course; since the order of the monodromy is 6, this simply re-enlarges the axion moduli space to have 2π period.

3.3 Incorporating string loop corrections

Since we can dial the size of the string coupling by a diligent choice of fluxes, we can meaningfully incorporate the first string loop corrections to our result. Recall that at the two derivative level, the vector multiplet moduli space receives no corrections at higher loops in g_s. The hypermultiplet moduli space receives both perturbative and non-perturbative corrections. The perturbative corrections in the case of a single hypermultiplet have been studied in e.g. [26–28].

Our main interest is whether the additional features introduced by the known loop corrections can qualitatively change the behavior of the potential in the region of Kähler moduli space where $A_1 > 0$; recall that in the absence of corrections, vacua at finite g_s only arise if $A_1 < 0$. We will find that the $A_1 > 0$ region in parameter space can be redeemed if two other inequalities, which depend sensitively on the Kähler *and* the flux data, are satisfied.

The potential takes the form

$$V = A_1 e^{2\phi} + A_2 e^{4\phi} + A_3 e^{6\phi}\,, \tag{3.10}$$

where

$$A_1 = -\Big((\mathrm{Im}\,\mathcal{N})^{-1\,00} + \frac{3}{4\mathcal{K}} X^0 \bar{X}^0\Big)(\tilde{p}+\tilde{q})^2 \tag{3.11}$$

$$A_2 = -(\mathrm{Im}\,\mathcal{N})^{-1\,IJ} a_I a_J + \chi_1\Big((\mathrm{Im}\,\mathcal{N})^{-1\,00} + \frac{1}{2\mathcal{K}} X^0 \bar{X}^0\Big)(\tilde{p}+\tilde{q})^2 \tag{3.12}$$

$$A_3 = \frac{\chi_1}{\mathcal{K}} X^I \bar{X}^J a_I a_J\,, \tag{3.13}$$

and $\chi_1 = \frac{4\zeta(2)\chi}{(2\pi)^3}$, where $\chi = 2$, the Euler number of our make-believe rigid Calabi-Yau. Note that we must distinguish between the terms in the potential stemming from the NSNS and the RR sector of string theory. The latter are kept to one higher order in $e^{2\phi}$.

Let's focus our attention on the string coupling dependence again. The minima of the potential with regard to g_s lie at

$$\frac{-A_2 \pm \sqrt{A_2^2 - 3A_1A_3}}{3A_3}\,, \tag{3.14}$$

if this is positive, else at 0. Again, our analysis boils down to the signs of the three coefficients A_i. A_3 is positive. The signs of A_1 and A_2 depend on where we are in the Kähler moduli space. First, let's consider the case for which minima away from vanishing string coupling existed at tree level, $A_1 < 0$. A quick glance at (3.14) convinces us that the same is true here, independent of the sign of A_2. Next, consider $A_1 > 0$. Unlike the situation at tree level, a minimum at finite string coupling is now possible in this region of parameter space, if the inequalities $A_2 < 0$ and $A_2^2 - 3A_1A_3 > 0$ can be satisfied. The first contribution to A_2 is positive definite. The sign of the second depends on where we are in Kähler moduli space, but $A_1 > 0$ implies that this term is negative. Hence, by choosing the magnitude of the fluxes $\tilde{p}, \tilde{q}$ carefully compared to that of n, e_1, we can arrange for $A_2 < 0$. Notice that unlike the case at tree level, the fluxes enter crucially in this analysis. Incorporating the second inequality into our considerations requires launching a numeric study of the Kähler data similar to our analysis of the tree level inequality above. We leave this to the interested reader.

The lesson we glean from this study is that, as expected, the computable quantum corrections to V allowed by $\mathcal{N} = 2$ supergravity give rise to interesting substructure in our potentials. Note also that, once one has incorporated the positive term A_3 into the potential (3.10), this toy model can potentially admit de Sitter vacua, which was not possible in the approximation of section 3.1.

4. Discussion

Our construction provides another illustration of the fact that fairly common string theory backgrounds can include enough different effects in the potential to stabilize the geometric moduli. Our toy model was not sufficiently complicated to admit parametric control; it would be interesting to find an analogue of the tuning parameter W_0 of [7], which would allow one to stabilize some small fraction of the models in the semi-classical regime. Since the unknown corrections to the potential in this class of models are down by more powers of M_s than in typical $\mathcal{N}=1$ constructions, but the potential is nevertheless a reasonably generic function of all moduli, one expects this class of models to be a good toy laboratory for studying moduli stabilization.

In the IIB context, it has recently become clear that the space of flux vacua admits a statistical description [31]. The class of IIA models we study in this paper seems even more amenable to such analysis, since the potential (2.2) relevant to compactification on a given Calabi-Yau manifold X can be completely constructed in terms of classical geometric data on X and its mirror manifold Y. In other words, this is a setting in which, in the leading approximation, mirror symmetry allows one to compute potentials for all moduli.

One should be able to generalize this construction to a class of $\mathcal{N}=1$ Calabi-Yau orientifold models where all geometric moduli enjoy flux-generated potentials. In that context, the leading approximation to the potential would still be given by gauged supergravity formulae (as in the type-IIB orientifolds). We note here that the negative contribution to the scalar potential coming from the inclusion of orientifold planes in the $\mathcal{N}=1$ setting should actually make the stabilization of moduli considerably simpler there than in the $\mathcal{N}=2$ constructions presented here (for discussions of the helpful role of orientifold planes, see [6] and [3]). Furthermore, there has recently been significant progress in both constructing semi-realistic chiral brane models, and in combining them with flux compactifications (see e.g. [32 − 36]). Many of these constructions arise in IIA string theory, and should naturally admit embeddings into (orientifolds of) our class of Calabi-Yau flux models.

Finally, the connection between gauged supergravity and string compactification fairly begs the question: which class of string compactifications is generic enough to yield the most general (or even most general abelian) gaugings imaginable? We leave these as promising directions for future work.

Acknowledgments

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A. The reduction

The starting point of our analysis is the following ten-dimensional type-IIA supergravity action,

$$S = \int e^{-2\hat{\phi}}\left(\frac{1}{2}R*1 + 2d\hat{\phi}\wedge *d\hat{\phi} - \frac{1}{4}H_3\wedge *H_3\right) - \frac{1}{2}(F_2\wedge *F_2 + \tilde{F}_4\wedge *\tilde{F}_4) + \\ +\frac{1}{2}H_3\wedge C_3\wedge dC_3 - B\wedge F_4^{\rm flux}\wedge dC_3\,, \tag{A.1}$$

where

$$H_3 = dB + H_3^{\rm flux}\,, \tag{A.2}$$

$$F_2 = dC_1\,, \tag{A.3}$$

$$F_4 = dC_3 + F_4^{\rm flux}\,, \tag{A.4}$$

$$\tilde{F}_4 = F_4 - C_1\wedge H_3\,. \tag{A.5}$$

As reviewed in section 2, the 4d effective theory of the compactified 10d SUGRA becomes gauged upon turning on fluxes. Gauged $\mathcal{N}=2$ SUGRA in 4d exhibits a potential which is completely determined in terms of the $\mathcal{N}=2$ data of the ungauged theory and the killing vectors of the isometries that are being gauged. Our goal in this appendix is to derive the potential as well as the killing vectors from dimensional reduction, and demonstrate that they fit the expected $\mathcal{N}=2$ mold.

A.1 Potential and killing vectors from reduction

We consider compactification on a Calabi-Yau X. We begin by decomposing the field strengths as follows,

$$F_4 = dc_3 + dA^i\omega_i + d\xi^A\alpha_A + d\tilde{\xi}_A\beta^A + e_i\tilde{\omega}^i \tag{A.6}$$

$$H_3 = db_2 + db^i\omega_i + p^A\alpha_A + q_A\beta^A\,. \tag{A.7}$$

Here, ω_i, $i=1,\ldots,h_{1,1}$, are a basis for $H^{1,1}(X)$, and $\tilde{\omega}^i$ a dual basis spanning $H^{2,2}(X)$, i.e.

$$\int_X \omega_i\wedge\tilde{\omega}^j = \delta_i^j\,. \tag{A.8}$$

α_A, β^A, $A=0,\ldots,h_{2,1}$, are a symplectic basis for $H^3(X)$,

$$\int_X \alpha_A\wedge\beta^B = \delta_A^B\,. \tag{A.9}$$

Contributions to the potential arise from the kinetic terms of C_3,

$$-\frac{1}{2}(\tilde{F}_4\wedge *\tilde{F}_4) \to -\frac{1}{8\mathcal{K}}e_ie_jg^{ij}\,, \tag{A.10}$$

and the kinetic term for B_2,

$$-\frac{e^{-2\hat{\phi}}}{4}H_3\wedge *H_3 \to -\frac{e^{-2\hat{\phi}}}{4}(p^Ap^B\alpha_A\wedge *\alpha_B + p^Aq_B(\alpha_A\wedge *\beta^B + \beta^B\wedge *\alpha_A) +$$

$$+ q_A q_B \beta^A \wedge *\beta^B) \,. \tag{A.11}$$

Upon expressing the integrals over the 3 cycles in terms of the period matrix $\mathcal{M}$, and introducing the 4d dilaton $e^{-2\phi} = e^{-2\hat{\phi}}\mathcal{K}$, this last expression becomes

$$\frac{e^{-2\phi}}{4\mathcal{K}}(q + p\mathcal{M})(\mathrm{Im}\,\mathcal{M})^{-1}(q + p\bar{\mathcal{M}}) \,. \tag{A.12}$$

In addition, the spacetime field c_3 turns out to play an important role in determining the potential. We collect all terms containing c_3,

$$\begin{aligned} &-\frac{1}{2}(\tilde{F}_4 \wedge *\tilde{F}_4) + \frac{1}{2}H_3 \wedge C_3 \wedge dC_3 - B \wedge F_4^{\rm flux} \wedge dC_3 \to \\ &\quad \to -\frac{\mathcal{K}}{2}(dc_3 - A \wedge db_2) \wedge *(dc_3 - A \wedge db_2) + (p^A\tilde{\xi}_A - q_A\xi^A - b^i e_i)dc_3 \,. \end{aligned} \tag{A.13}$$

A here is the 4d graviphoton field which descends from C_1. dc_3, being dual to a 0 form field strength, carries no local degrees of freedom. We can eliminate it from the action, following [37, 19], by solving for it via its equations of motion and plugging back into the action,

$$-\int P(dc_3 - j) \wedge *(dc_3 - j) + Q\, dc_3 \to -\int \frac{1}{4P}(Q+n)^2 + (Q+n)j \,. \tag{A.14}$$

Here, n is an integration constant which must be chosen to be integral [38]. This choice of integer in the potential is the exact analogue of the integer appearing in the potential of the massive Schwinger model studied by Coleman [39, 37]. The part of codimension 2 objects charged under the top form which are nucleated out of the vacuum is here played by $D2$ branes in spacetime. Since F_6 flux jumps in between a $D2/\,\overline{D2}$ pair, we see that the choice of n from a 10d point of view corresponds to the choice of F_6 flux.

Returning to our task, we apply the above to (A.13),

$$-\frac{1}{2\mathcal{K}}(-b^i e_i + p^A\tilde{\xi}_A - q_A\xi^A + n)^2 + (-b^i e_i + p^A\tilde{\xi}_A - q_A\xi^A + n)A \wedge db_2 \,. \tag{A.15}$$

Collecting terms and passing to the Einstein frame, $\sqrt{g} \to \sqrt{g}e^{4\phi}$, we arrive at the 4d potential

$$V = \frac{e^{4\phi}}{2\mathcal{K}}\left(\frac{1}{4}g^{ij}e_ie_j + (n - b^ie_i + p^A\tilde{\xi}_A - q_A\xi^A)^2\right) - \frac{e^{2\phi}}{4\mathcal{K}}(q + p\mathcal{M})(\mathrm{Im}\,\mathcal{M})^{-1}(q + p\bar{\mathcal{M}}) \,. \tag{A.16}$$

Next, we turn towards determining the killing vectors of the isometries being gauged. These can be read off from the covariantized kinetic terms of the 4d fields which acquire charges under gauging. To obtain the kinetic term for the axion $da = *db_2$, let us collect all terms involving b_2,

$$\begin{aligned} &-\frac{e^{-2\phi}\mathcal{K}}{4}db_2 \wedge *db_2 + \\ &+(-b^ie_i + p^A\tilde{\xi}_A - q_A\xi^A + n)A \wedge db_2 + \end{aligned}$$

$$
\begin{aligned}
+\frac{1}{2}db_2\wedge(\tilde{\xi}_A d\xi^A-\xi^A d\tilde{\xi}_A+2e_iA^i) \;=\; & -\frac{e^{-2\phi}\mathcal{K}}{4}db_2\wedge *db_2 \\
& +\frac{1}{2}db_2\wedge[(2b^ie_i-2n-p^A\tilde{\xi}_A+q_A\xi^A)A+ \\
& \qquad +2e_iA^i+\tilde{\xi}_A(d\xi^A-p^AA)- \\
& \qquad -\xi^A(d\tilde{\xi}_A-q_AA)]\,.
\end{aligned}
$$

Dualizing b_2 along the lines of

$$
-\int[P(db_2\wedge *db_2)-\frac{1}{2}db_2\wedge j]\to-\int\frac{1}{16P}(da+j)\wedge *(da+j) \tag{A.17}
$$

yields

$$
\begin{aligned}
&-\frac{e^{2\phi}}{4\mathcal{K}}[da+(2b^ie_i-2n-p^A\tilde{\xi}_A+q_A\xi^A)A+ \qquad\qquad (A.18)\\
&+2e_iA^i+\tilde{\xi}_A(d\xi^A-p^AA)-\xi^A(d\tilde{\xi}_A-q_AA)]^2 \;=\; -\frac{e^{2\phi}}{4\mathcal{K}}[Da+\tilde{\xi}_AD\xi^A-\xi^AD\tilde{\xi}_A]^2
\end{aligned}
$$

with

$$
\begin{aligned}
Da &= da+(2b^ie_i-2n-p^A\tilde{\xi}_A+q_A\xi^A)A+2e_iA^i\,, && (A.19)\\
D\xi^A &= d\xi^A-p^AA\,, && (A.20)\\
D\tilde{\xi}_A &= d\tilde{\xi}_A-q_AA\,. && (A.21)
\end{aligned}
$$

Hence, we see that, as promised in section 2, turning on H_3 and F_4 leads to the gauging of the following isometries,

$$
\begin{aligned}
k_0^a &= 2n-2b^ie_i+p^A\tilde{\xi}_A-q_A\xi^A\,, && (A.22)\\
k_0^{\xi^A} &= p^A\,,\\
k_0^{\tilde{\xi}_A} &= q_A\,,\\
k_i^a &= -2e_i\,. && (A.23)
\end{aligned}
$$

A.2 Consistency with gauged SUGRA

The most general scalar potential in $\mathcal{N}=2$ gauged supergravity with only electric charges is given by

$$
V=e^KX^I\bar{X}^J(g_{i\bar{j}}k_I^ik_J^{\bar{j}}+4h_{uv}k_I^uk_J^v)-(\frac{1}{2}(\mathrm{Im}\mathcal{N})^{-1\,IJ}+4e^KX^I\bar{X}^J)\mathcal{P}_I^x\mathcal{P}_J^x\,. \tag{A.24}
$$

Let's take a closer look at the various ingredients in turn. $g_{i\bar{j}}$, $i,\bar{j}=1,\dots,n_V$, and h_{uv}, $u,v=1,\dots,4n_H$, are the metrics on the vector and hypermultiplet moduli space, $\mathcal{M}_V$ and $\mathcal{M}_H$, respectively. The tree level metric on $\mathcal{M}_H$ was derived in [40] from dimensional reduction to be

$$
ds^2=d\phi\otimes d\phi+g_{ab}dz^a\otimes dz^b+\frac{e^{4\phi}}{4}\left[da+\tilde{\xi}_Ad\xi^A-\xi^Ad\tilde{\xi}_A\right]\otimes\left[da+\tilde{\xi}_Ad\xi^A-\xi^Ad\tilde{\xi}_A\right]-
$$

$$-\frac{e^{2\phi}}{2}(\mathrm{Im}\,\mathcal{M}^{-1})^{AB}\left[d\tilde{\xi}_A+\mathcal{M}_{AC}d\xi^C\right]\otimes\left[d\tilde{\xi}_B+\overline{\mathcal{M}}_{BD}d\xi^D\right].$$

k_I^i, $I=0,\ldots,n_V$, are the components of killing vectors encoding isometries of $\mathcal{M}_V$, gauged by the gauge field from the Ith vector multiplet, $I=0$ denoting the graviphoton of the gravity multiplet. Due to $\mathcal{N}=2$ supersymmetry, gauging isometries of $\mathcal{M}_V$ implies introducing non-abelian gauge symmetries. These isometries hence cannot become gauged merely by turning on fluxes. The k_I^u are the killing vectors on $\mathcal{M}_H$ we determined above, and the $\mathcal{P}_I^x$ are the corresponding prepotentials, which we shall compute for the case of $n_H=1$ in the next subsection. For the analysis immediately below, we will only need to know $\mathcal{P}_i^x=-e^{2\phi}e_i\delta^{x3}$.

$\mathrm{Im}\,\mathcal{N}$ is the gauge coupling matrix and $\mathrm{Re}\,\mathcal{N}$ the θ angle matrix for the vectors and graviphoton. When the metric on the Kähler moduli space is encoded in a prepotential $\mathcal{F}$, $\mathcal{N}$ is related to $\mathcal{F}$ by

$$\mathcal{N}_{IJ}=\bar{\mathcal{F}}_{IJ}+2i\frac{(\mathrm{Im}\,\mathcal{F})_{IK}X^K(\mathrm{Im}\,\mathcal{F})_{JL}X^L}{X^K(\mathrm{Im}\,\mathcal{F})_{KL}X^L}\,. \tag{A.25}$$

The standard large radius prepotential $\mathcal{F}=-\frac{1}{3!}\frac{\mathcal{K}_{ijk}X^iX^jX^k}{X^0}$ on the complexified Kähler moduli space arises as the vector multiplet prepotential upon dimensional reduction of the SUGRA action, *not* as we have written it down in (A.1), but after the field redefinition $C_3\to C_3-A_1\wedge B$. This new basis for the gauge fields is hence more natural when working with a prepotential. The gauge fields in the action (A.1) are related to those in this basis by $A_i\to A_i-b_iA$. Under this transformation, the killing vectors (A.22) transform into

$$k_0^a = 2n+p^A\tilde{\xi}_A-q_A\xi^A\,, \tag{A.26}$$

$$\begin{aligned}
k_0^{\xi^A} &= p^A\,,\\
k_0^{\tilde{\xi}_A} &= q_A\,,\\
k_i^a &= -2e_i\,.
\end{aligned} \tag{A.27}$$

Since we will be working in this new basis from now on, we do not bother with introducing new notation to distinguish the two bases. The advantage of using the transformed killing vectors is that when moving away from the large radius limit, we can simply use the period matrix of the mirror CY as our gauge coupling matrix, without need of transforming into a different basis.

From the prepotential given above, one easily obtains the following inverse gauge coupling matrix [19],

$$(\mathrm{Im}\,\mathcal{N})^{-1}=-\frac{1}{\mathcal{K}}\begin{pmatrix}1 & b^i\\ b^j & \frac{1}{4}g^{ij}+b^ib^j\end{pmatrix}, \tag{A.28}$$

where $e^{-K}=8\mathcal{K}$.

With these preparations, we now demonstrate that the potential (A.16) is consistent with the general form of the potential (A.24) obtained from gauged supergravity. For readability, we divide the potential into three pieces and evaluate them separately.

Terms involving only the graviphoton killing vectors

$$
\begin{aligned}
\frac{1}{2\mathcal{K}} h_{uv} k_0^u k_0^v &= \frac{1}{2\mathcal{K}} \left[\frac{e^{4\phi}}{4} (k_0^a + \tilde{\xi}_A k_0^{\xi^A} - \xi^A k_0^{\tilde{\xi}_A})^2 - \frac{e^{2\phi}}{2} (q + p\mathcal{M})(\mathrm{Im}\ \mathcal{M})^{-1} (q + p\bar{\mathcal{M}}) \right] \\
&= \frac{1}{2\mathcal{K}} \left[\frac{e^{4\phi}}{4} (2n + p^A \tilde{\xi}_A - q_A \xi^A + \tilde{\xi}_A p^A - \xi^A q_A)^2 - \right. \\
&\qquad \left. - \frac{e^{2\phi}}{2} (q + p\mathcal{M})(\mathrm{Im}\ \mathcal{M})^{-1} (q + p\bar{\mathcal{M}}) \right] \\
&= \frac{1}{2\mathcal{K}} \left[e^{4\phi} (n + p^A \tilde{\xi}_A - q_A \xi^A)^2 - \frac{e^{2\phi}}{2} (q + p\mathcal{M})(\mathrm{Im}\ \mathcal{M})^{-1} (q + p\bar{\mathcal{M}}) \right] . \quad \text{(A.29)}
\end{aligned}
$$

Terms involving only the vector multiplet killing vectors

$$
\begin{aligned}
4e^K X^i \bar{X}^j (h_{uv} k_i^u k_j^v - P_i^x P_j^x) - \frac{1}{2} (\mathrm{Im}\ \mathcal{N})^{-1\, ij} P_i^x P_j^x &= -\frac{1}{2} (\mathrm{Im}\ \mathcal{N})^{-1\, ij} P_i^x P_j^x \\
&= \frac{e^{4\phi}}{2\mathcal{K}} (\frac{1}{4} g^{ij} + b^i b^j) e_i e_j \, . \qquad \text{(A.30)}
\end{aligned}
$$

Terms mixing graviphoton and vector multiplet killing vectors

$$
\begin{aligned}
e^K (X^i \bar{X}^0 + X^0 \bar{X}^i) 4 h_{uv} k_0^u k_i^v \ &- \ [(\mathrm{Im}\ \mathcal{N})^{-1\, i0} + 4e^K (X^i \bar{X}^0 + X^0 \bar{X}^i)] P_i^x P_0^x \\
&= e^K (2b^i) 4 h_{uv} k_0^u k_i^v - [(\mathrm{Im}\ \mathcal{N})^{-1\, i0} + 4e^K (2b^i)] P_i^x P_0^x \\
&= \frac{1}{\mathcal{K}} b^i h_{uv} k_0^u k_i^v \\
&= -\frac{e^{4\phi}}{\mathcal{K}} (n + p^A \tilde{\xi}_A - q_A \xi^A) b^i e_i \, . \qquad \text{(A.31)}
\end{aligned}
$$

Adding the three contributions (A.29), (A.30), and (A.31), we arrive, as promised, at the potential (A.16) obtained by dimensional reduction.

B. The killing prepotential for the universal hypermultiplet

The hypermultiplet moduli space $\mathcal{M}_H$ is a quaternionic manifold of dimension $4n_H$. The structure group of the tangent bundle is hence the product $\mathrm{Sp}(2) \times \mathrm{Sp}(2n_H)$, and the Levi-Civita connection and curvature decompose accordingly. Relevant for defining the killing prepotential are the $\mathrm{Sp}(2) \cong \mathrm{SU}(2)$ connection, ω, and its curvature, Ω. In terms of these, the relation between a killing vector $k = k^a \partial_{q^a}$ encoding an isometry of $\mathcal{M}_H$ and the corresponding killing prepotential $\mathcal{P} = \mathcal{P}^x \sigma_x$ is

$$
\iota_k \Omega^x = D\mathcal{P}^x + \epsilon^{xyz} \omega^y \mathcal{P}^z \, . \qquad \text{(B.1)}
$$

Given the metric on $\mathcal{M}_H$ and a killing vector k, one must hence calculate the connection and curvature, extract the $\mathrm{Sp}(2)$ factor and then solve the above differential equation to determine the corresponding killing prepotential $\mathcal{P}$.

For simplicity, we will restrict ourselves to the universal hypermultiplet. The holonomy can be made explicit with the choice of vierbein

$$
q = \begin{pmatrix} u & v \\ \bar{v} & -\bar{u} \end{pmatrix} , \qquad \text{(B.2)}
$$

in terms of which the metric is given by

$$ds^2 = q^{A\alpha}(\sigma_2)_{AB}(\sigma_2)_{\alpha\beta}q^{B\beta}\,. \tag{B.3}$$

The 1-forms u and v get the following contributions at tree and 1-loop level [28]

$$u_0 = e^{\phi}(dx + idy) \tag{B.4}$$

$$v_0 = -d\phi + e^{2\phi}i\left(ydx - xdy + \frac{1}{2}da\right) \tag{B.5}$$

$$u_1 = -\chi_1 e^{2\phi}u_0 \tag{B.6}$$

$$v_1 = -\frac{1}{2}\chi_1 e^{2\phi}v_0\,, \tag{B.7}$$

where $\chi_1 = \frac{4\zeta(2)\chi}{(2\pi)^3}$, χ being the Euler number of the compactification manifold. The (first) Sp(2) factor of the curvature is given by [28]

$$\Omega_0^1 = i(\bar{u_0}\wedge v_0 + \bar{v_0}\wedge u_0) \tag{B.8}$$

$$\Omega_0^2 = (\bar{u_0}\wedge v_0 - \bar{v_0}\wedge u_0) \tag{B.9}$$

$$\Omega_0^3 = i(\bar{u_0}\wedge u_0 - \bar{v_0}\wedge v_0) \tag{B.10}$$

$$\Omega_1^1 = e^{2\phi}\left(-\chi_1\Omega_0^1 + i\frac{\chi_1}{2}(u_0\wedge v_0 - \bar{u_0}\wedge \bar{v_0})\right) \tag{B.11}$$

$$\Omega_1^2 = e^{2\phi}\left(-\chi_1\Omega_0^2 - \frac{\chi_1}{2}(u_0\wedge v_0 + \bar{u_0}\wedge \bar{v_0})\right) \tag{B.12}$$

$$\Omega_1^3 = e^{2\phi}\left(-2\chi_1\Omega_0^3 - 2i\chi_1(\bar{v_0}\wedge v_0)\right). \tag{B.13}$$

The three killing vectors of this metric that correspond to shift symmetries of the axions are

$$\mathbf{k_1} = \partial_a \tag{B.14}$$

$$\mathbf{k_2} = 2y\partial_a + \partial_x \tag{B.15}$$

$$\mathbf{k_3} = -2x\partial_a + \partial_y\,. \tag{B.16}$$

Note that these shift symmetries are preserved by the perturbative loop corrections to the metric (in fact, this is a crucial ingredient in deriving these corrections). To calculate the killing prepotential, we use the relation

$$\mathcal{P}^x = \frac{1}{4}D^i k^j \Omega^x_{ij}\,. \tag{B.17}$$

This yields

$$\mathcal{P}_1 = \frac{1}{2}\begin{pmatrix} 0 \\ 0 \\ e^{2\phi} - 4\chi_1^2 e^{6\phi}\end{pmatrix} \tag{B.18}$$

$$\mathcal{P}_2 = 2\begin{pmatrix} 0 \\ -e^{\phi} + \frac{1}{2}\chi_1 e^{3\phi} \\ e^{2\phi}y - 4\chi_1^2 y e^{6\phi}\end{pmatrix} \tag{B.19}$$

$$\mathcal{P}_3 = -2 \begin{pmatrix} e^{\phi} - \frac{1}{2}\chi_1 e^{3\phi} \\ 0 \\ e^{2\phi}x + 4\chi_1^2 x e^{6\phi} \end{pmatrix} . \tag{B.20}$$

To relate these results to our compactification, we need to identify the coordinates in which we obtain the quaternionic manifold via dimensional reduction to those introduced here. This is easily accomplished by comparing the metric (B.3) at tree level to the one obtained via reduction. The latter is

$$ds^2_{\rm red} = d\phi^2 + \frac{e^{4\phi}}{4}[da + \tilde{\xi}d\xi - \xi d\tilde{\xi}]^2 - \frac{e^{2\phi}}{2\mathcal{M}_I}[d\tilde{\xi}^2 + \mathcal{M}_R(d\xi d\tilde{\xi} + d\tilde{\xi}d\xi) + |\mathcal{M}|^2 d\xi^2] . \tag{B.21}$$

Recall that $\mathcal{M}$ is the period matrix on the complex structure moduli space, and in (B.21) we use the notation that $\mathcal{M}_{R,I}$ are its real and imaginary parts. Since we are considering a rigid Calabi-Yau, $\mathcal{M}$ is simply a constant in our example which encodes the expansion coefficients of the Hodge duals $*\alpha$ and $*\beta$ of a symplectic basis $\{\alpha, \beta\}$ of H^3 of the rigid CY in this basis. In particular,

$$(\operatorname{Im}\mathcal{M})^{-1} = -\int \beta \wedge *\beta , \tag{B.22}$$

i.e. $\operatorname{Im}\mathcal{M} < 0$. Plugging in the tree level expressions for u and v into (B.3) yields

$$ds^2 = d\phi^2 + e^{4\phi}4(da + 2(ydx - xdy))^2 + e^{2\phi}(dx^2 + dy^2) . \tag{B.23}$$

We can read off that $\sqrt{2}\binom{x}{y}$ and $\binom{\xi}{\tilde{\xi}}$ are related via a symplectic transformation. With the parametrization

$$\sqrt{2}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} \xi \\ \tilde{\xi} \end{pmatrix} , \tag{B.24}$$

we determine this transformation to be

$$\alpha = -\frac{|\mathcal{M}|}{\sqrt{-\mathcal{M}_I}} \tag{B.25}$$

$$\beta = -\frac{M_R}{|\mathcal{M}|\sqrt{-\mathcal{M}_I}} \tag{B.26}$$

$$\gamma = 0 \tag{B.27}$$

$$\delta = \frac{\sqrt{-\mathcal{M}_I}}{|\mathcal{M}|} . \tag{B.28}$$

This field identification remains correct at 1-loop level. By the linearity of (B.17), expanding the isometries gauged by the graviphoton (2.13) and the vectors in the vector multiplets (2.14) in this set of killing vectors allows us to read off the corresponding killing prepotentials. The expansions are easily determined to be

$$\mathbf{k_{grav}} = (2n + p\tilde{\xi} - q\xi)\partial_a + p\partial_\xi + q\partial_{\tilde{\xi}} \tag{B.29}$$

$$= 2n\mathbf{k_1} + \frac{1}{\sqrt{2}}(\alpha p + \beta q)\mathbf{k_2} + \frac{1}{\sqrt{2}}\delta q \mathbf{k_3} , \tag{B.30}$$

$$\mathbf{k_{vect}} = -2e\mathbf{k_1} . \tag{B.31}$$

C. Calculations at the Landau-Ginzburg point

For our example, we choose the CY given as a hypersurface in the weighted projective space $W\mathbb{P}^4_{2,1,1,1,1}$. The periods around the Landau-Ginzburg (LG) point can be determined in terms of the power series

$$\omega_0(\psi) = -\frac{1}{k\pi^4}\sum_{n=1}^{\infty}\frac{\prod_{i=0}^{4}\Gamma(\frac{n}{k}\nu_i)\sin(\frac{\pi n}{k}\nu_i)}{\Gamma(n)}\frac{e^{i\frac{\pi}{k}(k-1)n}}{\sin(\frac{\pi n}{k})}(\gamma\psi)^n\,, \tag{C.1}$$

where the ν_i are the weights of the ambient weighted projective space, $k = 6$ (this is the smallest common multiple of the powers n_i in the polynomial $\sum_{i=0}^4 x_i^{n_i}$ defining the hypersurface; e.g. $k = 5$ for the quintic), and $\gamma = k\prod_{i=0}^4(\nu_i)^{-\nu_i/k}$. A basis for the solutions to the Picard-Fuchs equations is now given by $\{\omega_0, \omega_1, \omega_2, \omega_5\}$, where $\omega_i(\psi) = \omega_0(\beta^j\psi)$ for $\beta = \exp(\frac{2\pi i}{k})$. Assembling these in a vector $\omega = -\frac{(2\pi)i^3}{OrdG}(\omega_2, \omega_1, \omega_0, \omega_5)^T$, they are related to a symplectic basis $\Pi' = (\mathcal{G}_1, \mathcal{G}_2, z^1, z^2)^T$ via the transformation $\Pi' = m\omega$, with m given by

$$m = \begin{pmatrix} -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 3 & 2 \\ 0 & 1 & -1 & 0 \end{pmatrix}. \tag{C.2}$$

To calculate the gauge coupling matrix, we need the second derivatives of the prepotential $\mathcal{G}$. Using the homogeneity of $\mathcal{G}$ these can be expressed in terms of the periods as follows,

$$\mathcal{G}_{12} = \frac{\mathcal{G}_1' - \frac{\mathcal{G}_1}{z^1}(z^1)'}{(z^2)' - \frac{z^2}{z^1}(z^1)'} \tag{C.3}$$

$$\mathcal{G}_{11} = \frac{\mathcal{G}_1}{z^1} - \mathcal{G}_{12}\frac{z^2}{z^1} \tag{C.4}$$

$$\mathcal{G}_{22} = \frac{\mathcal{G}_2}{z^2} - \mathcal{G}_{12}\frac{z^1}{z^2}\,, \tag{C.5}$$

where the prime denotes differentiation with regard to ψ. We next need to relate these results to the large radius limit. With the period vector in the large radius limit given as $\Pi \sim (t^2, t^3, t, 1)^T$, our choice of the matrix N relating the two bases (recall that this matrix is only specified up to monodromy) is given by

$$N = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{C.6}$$

After calculating the gauge coupling matrix in the basis Π', we can transform it to the basis Π: given a symplectic transformation $N = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$, the gauge coupling matrix $\mathcal{N}$ transforms as $\mathcal{N}' = \big(B + A\mathcal{N}(\psi)\big)\big(D + C\mathcal{N}(\psi)\big)^{-1}$.

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